## Linear Geometry in Space

- $|\vec{a} \times \vec{b}|$  is the area of the parallelogram formed by  $\vec{a}$  and  $\vec{b}$ .
- Triple Scalar Product:  $\vec{a} \cdot (\vec{b} \times \vec{c})$

• Let 
$$\vec{a} = \langle a_1, a_2, a_3 \rangle, \vec{b} = \langle b_1, b_2, b_3 \rangle$$
, and  $\vec{c} = \langle c_1, c_2, c_3 \rangle$ .  
•  $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ 

- Represents the volume of the **parallelopiped** formed by  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$ .
- The tetrahedron formed by  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  has one-sixth of this volume.
- **Lines** in three-space:
  - Let  $\vec{r}_o$  be the position vector of a point on a line  $\vec{r}$  in three-space, and let the direction vector  $\vec{L}$  be parallel to  $\vec{r}$ .
  - Parametric equation:  $\vec{r}(t) = \vec{r}_o + t\vec{L}$
  - Symmetric equation:  $\frac{x x_o}{a} = \frac{y y_o}{b} = \frac{z z_o}{c}$ 
    - Note: Three variables and two constraints. One degree of freedom.
- **Planes** in three-space:
  - Let  $\vec{r}_o$  be the position vector of a point on a plane in three-space, and let the vectors  $\vec{a}$  and  $\vec{b}$  lie in the plane.
  - $\circ$   $\vec{n} = \vec{a} \times \vec{b} = \langle a, b, c \rangle$  is normal to the plane
  - Vector equation:  $\vec{r}(s,t) = \vec{r}_o + s\vec{a} + t\vec{b}$
  - Parametric equation:  $\vec{r}(s,t) = \langle x(s,t), y(s,t), z(s,t) \rangle$
  - ax + by + cz = d, where  $\vec{r}_o \cdot \vec{n} = d$  and  $\vec{n} = \langle a, b, c \rangle$ 
    - Note: Three variables and one constraint. Two degrees of freedom.
- Basic Matrices and Linear Algebra:
  - Very useful in representing linear transformations.
    - This is seen in change of variables when transformations are needed.
  - Any systems of linear equations can be appropriately written as a matrix
  - Reduced-row echelon form
  - Perform **Gaussian elimination** to find the reduced-row echelon form of a matrix to find the intersection of planes (i.e. solving the system of linear equations).
  - Determinant
    - Only applies to square matrices
    - Useful in computing cross products and triple scalar products
    - Very useful in multivariable analysis